Advanced Level Geometry #1 — Triangles

Start at whatever problem you feel comfortable with. The questions are in a rough order of difficulty, and group work is encouraged as long as it does not disrupt the teacher. If you already understand the concepts, there are also challenge problems in the end.

1. △ABC is similar to △PQR. The ratio of their side lengths is 2:3, respectively. If △ABC has an area of 12 km^2, what is the area of △PQR?

2. Points A, B, C, D are chosen in the plane such that segments AB, BC, CD, DA have lengths 2, 7, 5, 12, respectively. What is the minimum and maximum possible value for AC?
    a. The *Triangle Inequality* states that the sum of the lengths of any two sides must be greater than or equal to the length of the remaining side.

3. A triangle with vertices (5,6), (7, -2), and (8,2) is reflected about the line x=7 to create a second triangle. What is the area of the union of the two triangles?

4. A triangle △ABC has AB=13, BC=14, and CA=15. If point D is on segment BC such that the area of △ABD is 6 times the area of △ACD, what is CD?

5. For a trapezoid ABCD, the diagonals AC and BD meet at point P. The point where the altitude drawn from point A intersects CD is point E. Given that DE = 5, AE = 12, AC = 20 and AB is ⅔ of the length of CD, find the area of triangle ADP.

Challenge Problems

1. The **Inscribed Angle Theorem** is a very useful formula relating inscribed angles in a circle. Consider a circle with Center O and choose three points A, B, and C on the circumference of the circle. Form the angle BAC, and BOC. The theorem relates the two angles, stating that BAC = (½) * BOC. To *prove* this theorem, there are three cases to consider:

    a. O is contained within the area enclosed by BAC.

    b. O lies on either BA or AC.

    c. O lies outside of the area enclosed by BAC.

    Considering these three cases, try to prove the theorem. *Hint:* Use isosceles triangles!