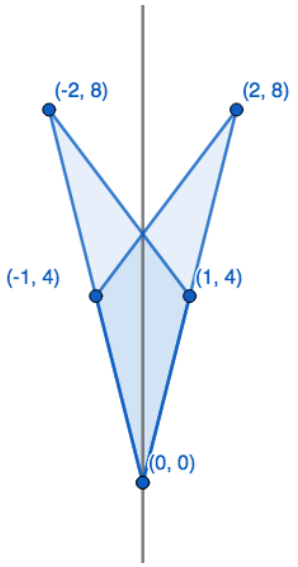
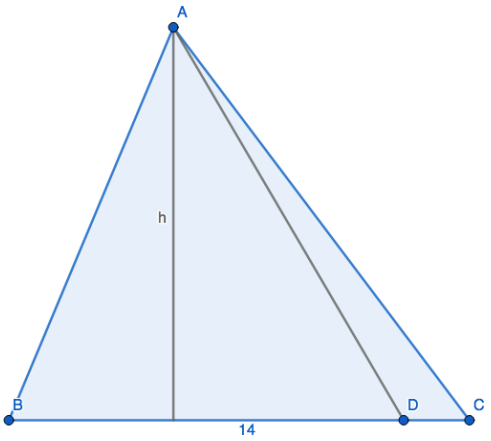
1. The area is proportional to the square of the side length, so $[PQR] = (3/2)^2[ABC] = 9/4 \cdot 12km^2 = 27km^2$
2. By the triangle inequality, $AC \leq AB + BC = 2 + 7 = 9$ and $AC \geq DA - CD = 12 - 5 = 7$, so the minimum possible value is 7 and the maximum is 9
3. We begin by translating every point 7 units to the left and 2 units up to get the following problem:

   A triangle with vertices (-2,8), (0, 0), and (1,4) is reflected about the line x=0 to create a second triangle. What is the area of the union of the two triangles?

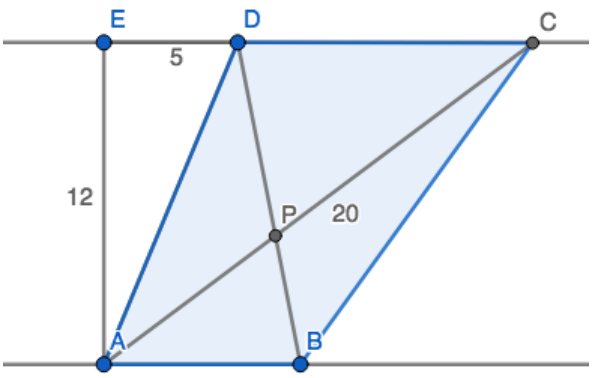
   

   It's easy to see that (0,0), (1,4), and (2,8) are collinear. Furthermore, the line through (-1,4) and (2,8) is $y = \frac{4}{3} \cdot x + \frac{16}{3}$, so the point of intersection of the three lines is (0, $\frac{16}{3}$). We want to find the area of the region bounded by (0,0), (1,4), (0, $\frac{16}{3}$), (-1,4). This is a kite with diagonals of length 2 and $\frac{16}{3}$, so the area is $\frac{1}{2} \cdot 2 \cdot \frac{16}{3} = \frac{16}{3}$.

4. By the formula $A = \frac{1}{2} \cdot bh$, is h is the length of the altitude of $\triangle ABC$, $[ABD] = \frac{1}{2}BD \cdot h$ and $[ACD] = \frac{1}{2}CD \cdot h$. We are given that $[ABD] = 6[ACD]$, and plugging in the values gives us $BD = 6CD$. Since $BD + CD = BC = 14$, we can solve these equations to find $CD = 2$.

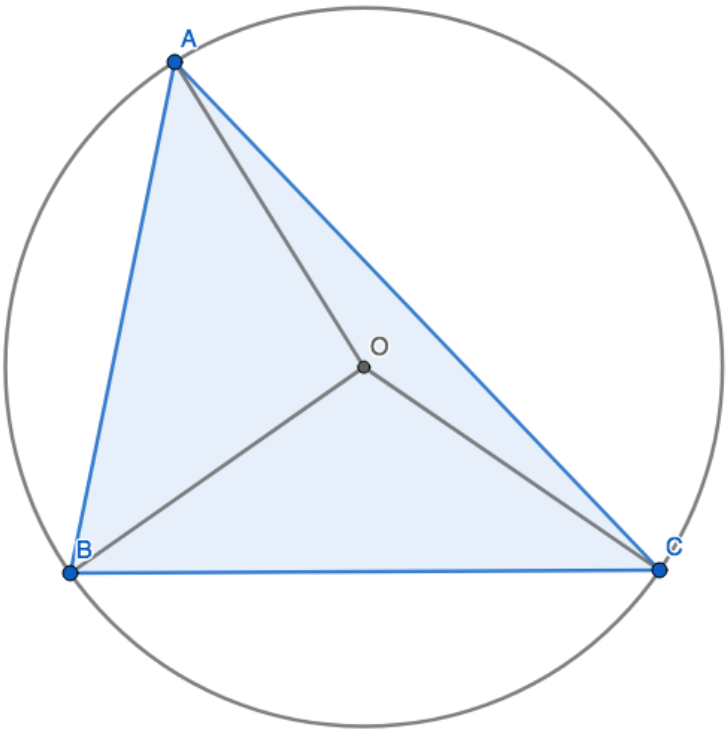
   

5. By the Pythagorean Theorem, $EC = \sqrt{20^2 - 12^2} = 18$, so $DC = 11$. It is given that $AB = \frac{2}{3} \cdot DC = \frac{22}{3}$. Since $ABCD$ is a trapezoid, AB and CD are parallel, so by vertical angles, $\angle ABD = \angle CDB$ and $\angle BAC = \angle DCA$, so by AA similarity,

$\triangle APB$ $\triangle CPD$. Let h_1 be the length of the height in $\triangle APB$ and let h_2 be the length of the height in $\triangle CPD$. By the similarity, $\frac{h_1}{2} = \frac{h_2}{3}$, and $h_1 + h_2 = EA = 12$. Solving these equations gives us $h_1 = \frac{24}{5}$. Using the formula $A = \frac{1}{2}bh$, $[APD] = [ABD] - [ABP] = \frac{1}{2} \cdot \frac{22}{3} \cdot 12 - \frac{1}{2} \cdot \frac{22}{3} \cdot \frac{24}{5} = \frac{132}{5}$

Challenge problem:

1. The proofs for the 3 cases are all very similar, so we will only present part (a). You can find the other parts at https://www.khanacademy.org/math/geometry/hs-geo-circles/hs-geo-inscribed-angles/a/inscribed-and-central-angles-proof.

   Since O is the center of the circle, OA=OB=OC. This means that △COA, △AOB, and △BOC are isosceles, meaning that
   $\angle BAC = \angle BAO + \angle CAO = \angle ABO + \angle ACO =$
   $\angle ABC - \angle CBO + \angle ACB - \angle BCO =$
   $(180 - \angle BCO - \angle CBO) - (180 - \angle ABC - \angle ACB) = \angle BOC - \angle BAC$
   , which rearranges to $\angle BOC = 2\angle BAC$