Number Theory #2 Solutions

1. We know that for a square number to occur, the power of any prime factor must be even. Thus we only consider the even powers for $2^{200}3^{199}5^{198}$ so we can divide by 2 to get $2^{100}3^{99}5^{99}$. We then multiply $101 * 100 * 100 = 1010000$.

2. We know that $72 = 2^3 3^2$. We can write this as $(1 + 2^3 + 2^6 + 2^9)(1 + 3^3 + 3^6) = 585 * 757$ $= 442845$.

3. The geometric series formula is $S = \frac{a(r^n - 1)}{r-1}$. We know the first term will always be 1 so we can eliminate the a. Thus, for a number $p_1{}^a * p_2{}^b * p_3{}^c ...$ the sum of factors will be $(p_1{}^{a+1} - 1)/(p_1 - 1) * (p_2{}^{b+1} - 1)(p_2 - 1) * (p_3{}^{c+1} - 1)(p - 1).....$

41.
(a) $2^{18}3^{12}$
(b) $2^{30}3^{15}$
(c) $2^{20}7^5$
(d) $7^2 13^2$
(e) $2^{24}5^{24}$
(f) $2^{30}5^6$

42. We know that $36 = 2^2 3^2$ so the sum is $(1 + 2 + 2^2)(1 + 3 + 3^2) = 7 * 13$. Since both are prime factors, we repeat the process of $(1 + 7)(1 + 13) = 8 * 14 = 112$.

43. a) The sum of the proper factors of 28 is $(1 + 2 + 4)(1 + 7) - 28 = 28$, thus 28 is a perfect number.
b) As prime numbers only have 2 factors, (1 and itself), the sum of proper factors must be equal to 1. So prime numbers are deficient, thus 7 and 13 are deficient.
For 26 we do $(1 + 2)(1 + 13) - 26 = 16$. Thus it is abundant.
For 18, the sum of proper factors of 18 is equal to $(1 + 3 + 9)(1 + 2) - 18 = 21$ so it is abundant.
For 30 the sum is $(1 + 5)(1 + 2)(1 + 3) - 30 = 42$ so it is abundant.
For 25 we have $1 + 5 = 6$ so it is deficient.
c) As prime numbers only have 2 factors, (1 and itself), the sum of proper factors must be equal to 1. So prime numbers are deficient.
d) We write the sum of the factors as $(1 + 2 + ... + 2^n)(1 + 2^{n+1} - 1) - 2^n(2^{n+1} - 1)$ which is equal to $(2^{n+1} - 1)(2^{n+1}) - (2^{n+1} - 1)(2^n) = (2^{n+1} - 1)(2^n)$, thus the number is perfect.